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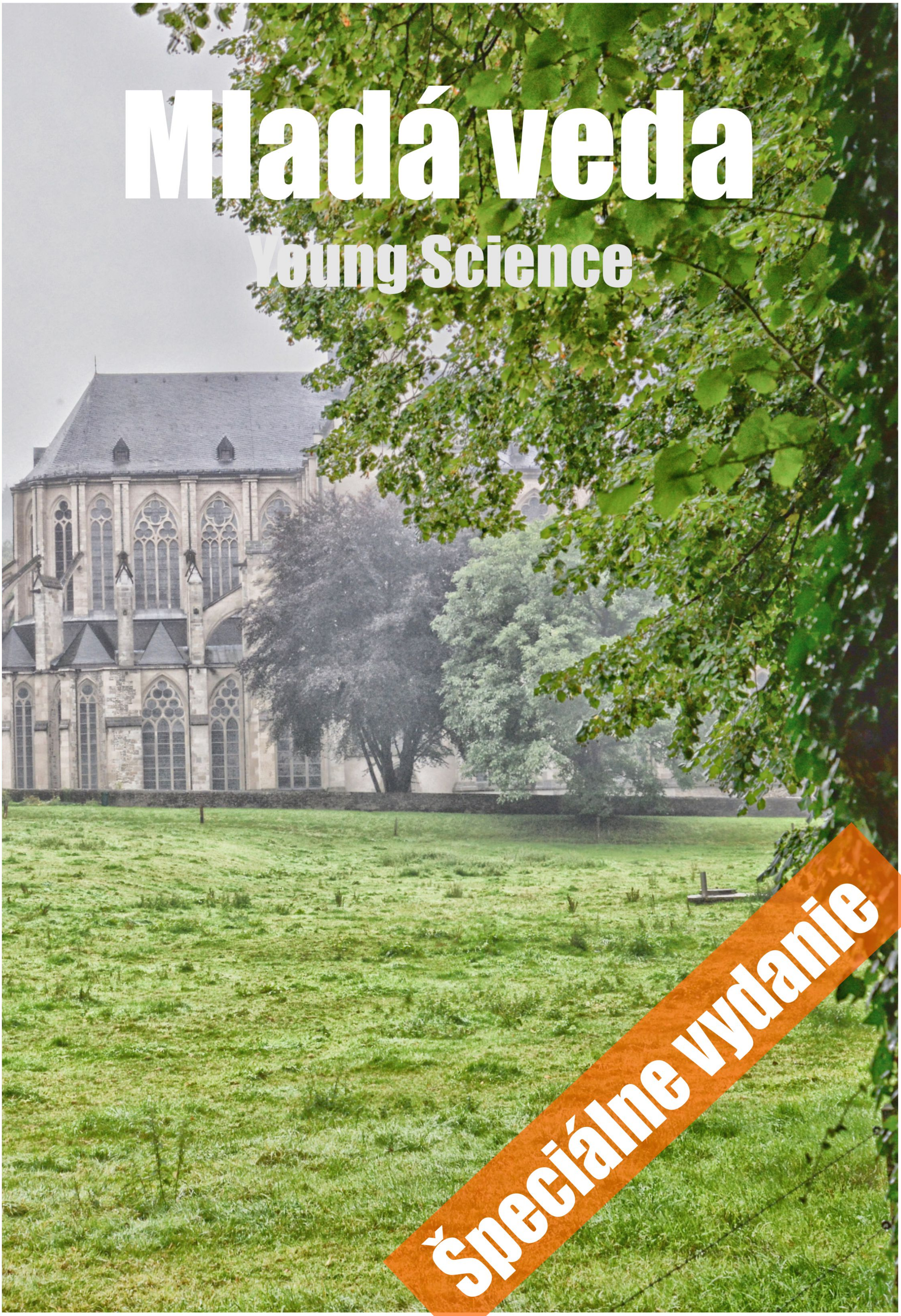

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HOW TO CREATE EXAMPLES FOR MULTIPLE-CHOICE TESTING IN MATHEMATICS

JAK TVOŘIT PŘÍKLADY V MATEMATICE PRO TESTY S VÍCE ODPOVĚĎMI

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Abstract
In the paper several aspects of the computer-adaptive testing in mathematics are discussed. The good choice of appropriate examples is studied. The selection on the examples is carried out from two different perspectives. The appropriate choice is illustrated on concrete examples.
Key words: computer-adaptive testing, multiple-choice, mathematics

Abstrakt
V článku je diskutováno několik aspektů elektronického testování v matematice. Dobrá volba vhodných příkladů je studována. Výběr příkladů je prováděn ze dvou různých hledisek. Příslušná volby je ilustrována konkrétními příklady.
Klíčová slova: elektronické testování, volba více odpovědí, matematika

Introduction

Currently, computer technology pervades very strongly everyday life and also teaching of all subjects. On one hand, the use of computer technology in mathematics supports deeper understanding of some subjects. On the other hand, it distracts attention from the simplest solution to using algorithms, which can complicate the solution process.

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This situation puts higher demands on all the university teachers (not just teachers of mathematics) and at the same time it is a great challenge for them. With the availability of electronic resources it is necessary to focus on aspects of teaching from different angles. *These include:*

• students' attitude to the subject (Vysoká & Smetanová(2016)),
• creation and quantity of teaching materials (Boháč et al (2016), Schreiberová & Volný (2017));
• factors influencing human reliability, including education (Vondráčková et al (2016)),
• other aspects (Boháč et al (2015), Pilous & Janda (2017) and others).

Involvement of computer technology into education takes place in various forms in greater or lesser extent, for example:

• problems of visualization (Klepancová & Smetanová (2015), Kmeťová (2015), Schreiberová & Volný (2017), Veide & Strozheva (2015));
• application to specific exercises (Biba et al (2016), Čejka et al (2017), Hlaváčová (2017), Vysoká (2015)),
• use in numerical methods calculations (Biba et al (2016)),
• preparing teaching materials for students (Schreiberová & Volný(2017));
• explaining mathematical problems in equivalent manner (Varga et al (2014) Vargová et al (2016)),
• promoting communication in education and simulation of social networking in education (Jelínek & Novotný (2014), Jelínek (2016)),
• computer-adaptive testing (especially in mathematical knowledge) (Chládek & Smetanová(2014), Chládek & Smetanová (2015), Chládek & Smetanová (2017), Říhová & Viskotová (2015), Schindler et al (2006), Smetanová (2015)).

In this article we will focus on electronic testing in mathematics and the creation of appropriate exercises for tests with the choice of the correct answer. Suitability of examples is being discussed from various perspectives.

Computer-adaptive testing in mathematics

Computer-adaptive testing is a modern method of verifying students' knowledge. In many other fields and disciplines it already is permanently and effectively used, mainly to save time. Possibilities of different types of tests are mentioned in the Schindler et al (2006). We will focus mainly on tests with selecting one or more correct answers which belong to tests with closed questions.

Mathematics, however, is perceived as traditional and conservative scientific field. Therefore, we can often meet the opinion that electronic tests in mathematics are not an appropriate instrument for testing students. We must remark that due to the nature of the field all the necessary knowledge can really not be tested by tests with the choice of the correct answers. In mathematics we can mention outlining graph of a function.

Still, computer-adaptive testing can be used in mathematics to test some partial knowledge. In our opinion, it is very well applicable for continuous testing of knowledge and in larger groups. This is mainly due to the huge time savings.

Advantages and disadvantages of closed questions tests are presented in Schindler et al (2006).

Advantages of close questions: (Schindler et al. (2006), pp. 36–37)
- the assessment is quick and in the electronic form may be automatized;
- the assessment is objective, it is possible to decide ambiguously if the given answer is correct or incorrect;
- close questions are suitable for students who struggle to formulate the answers and are slow at writing,
- the answer is not dependent on the expression abilities of the students and their graphomotoric capacities – very often, students only circle or tick the right answers.

Disadvantages of close questions: (Schindler et al. (2006), pp. 37–38)
- some skills are of a productive nature (e. g. to formulate an opinion, to sketch a picture, graph or diagram etc.) which cannot be tested with the use of close questions or only to a limited extent;
- it handicaps students inattentive or distracted students (they may overlook between alternative answers or they make a mistake when ticking the right answer), or contemplative students who think deeply over the alternative answers which makes them unsure or who try to identify a trick answer;
- there is certain probability that students tip the right answers, which rises a small number of alternatives (2–3) ,
- it is not possible to track the thinking process of the students;
- the creation of a good close question is not easy, the revelation of construction deficiencies is not always clear from students' answers and it requires a deeper analysis;
- students may copy more easily;
- the community of teachers (at least in the Czech Republic) have a rather sceptical attitude to close questions and they are used to a limited extent in the teaching and learning process.

How the advantages and disadvantages mentioned above manifest themselves in mathematics is discussed in Chládek & Smetanová (2017).
It is important to realize that the basis for electronic testing is a suitable software application. The application should enable creating test questions, offer them to the students within the specified time to answer and evaluate the test results.
Such application is a part of the school information system at the Institute of Technology and Business in České Budějovice. This application enables testing of students with different kinds of tests. Tests with closed questions are probably used most by teachers of foreign languages. They also appear at interim or final testing in various vocational subjects. They are used in a lesser extent in mathematics than in other subjects, especially for continuous testing during the semester for students of combined form.

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The experience with several years of testing mathematical skills (over 600 students since 2013) are described in Chládek & Smetanová (2014), Chládek & Smetanová (2015), Chládek & Smetanová (2017) Smetanová (2015).
The probability of guessing the correct answers in this type of tests and how to reduce the likelihood is discussed in Chládek & Smetanová (2014), Chládek & Smetanová (2015).

How to create examples in mathematics

In this chapter we will show how to create examples for computer-adaptive testing. We will not deal directly with examining theoretical knowledge in mathematics. The tests will be focused on the ability to solve arithmetic tasks and on applying theoretical knowledge in solving specific examples.
Developing the examples for electronic testing of mathematics can be approached in several ways. First, we have to choose which parts of the student's mathematical skills we want to focus on. It is possible to test the ability to apply mathematical calculus or application of theoretical knowledge to exercises. Both skills (applying calculus and application of theoretical knowledge) are represented in varying degrees in solving all problems. We can strengthen the application of one of them by appropriate choice of the exercise.
We will consider two types of examples for electronic testing: *a) calculus* (the calculation must be performed for the correct answer), *b) reasoning* (good knowledge of theory is enough to solve the examples).

Calculus:

If we want to focus on making the student calculate the exercise, we can choose straightforward exercises. Exercises with one choice answers are best suited for this strategy. The most common structure of the exercise is the question for the result.
The following examples represent possible choice of exercises which require calculation and it is difficult to apply only simple reasoning.

Example A:

Calculate $\int_0^9 \left(\sqrt{x}+3\right) dx$.

Choose the answer: a) 27, b) $\frac{4}{3}$, c) 0, d) $\frac{\sqrt{3}}{2}$, e) no of the other answers is correct.

Example B:

Calculate $\int \ln(x^2+1)\, dx$.

Choose the answer: a) $x\ln(x^2+1)-2\arctan x+2x+C, C\in R$, b) no of the other answers is correct, c) $(x^2+1)\ln x-2\arctan x+2x+C, C\in R$, d) $x\ln(x^2+1)+2\arctan x+2x+C, C\in R$.

Example C:

Find all solutions $x+2y+3z=4$, $2x+y-\ z=3$, $3x+3y+2z=7$.

Choose the answer: a) $x=\frac{2}{3}, y=\frac{5}{3}, y=0$, b) $x=\frac{21}{3}, y=\frac{5}{3}, y=3$, c) no of the other answers is correct, d) $x=\frac{8}{3}, y=-\frac{2}{3}, y=1$.

When selecting this type of example we need to pay attention to the fact which mathematic skills we want to test. The Examples A and B appear to be intended to verify knowledge of calculus of integrations. The solution is to calculate definite respectively indefinite integral and compare the result. Example B is very demanding of attention, only to overlook the sign can lead to a bad result.

Despite the fact that Example B requires finding primitive function, students can solve it differently. It is enought o realize that derivation of primitive function is a function occurring in the integration (inverse relationship between derivation and integration). All functions can be derivated and thus the students can find the correct result. If they "hit" the function properly and derivate it, the results can be found very quickly.

Calculating derivative functions is for students often much easier than calculating integrals. It is hard to exactly determine which mathematical skills the students used in solving this example. The example can test the knowledge of integration, but also the knowledge of the derivative compound function.

Example C appears to be similar in nature as example B. It appears that the solution can be bypassed by the system of linear equations. It is not so. It is enough to substitute and we find that a) is the solution of the equation. However, a) is not the correct answer. The question asks about all solutions and this system has infinitely many of them. Answer a) describes only one of them. The correct answer (surprising for many students) is c).

Benefits of the calculus type:

• verification of arithmetic skills,

• use of this type is very useful as preparation for the final written test,

• suitable for continuous check of students' arithmetic skills,

• quick computer evaluation, saving time for the teacher.

Disadvantages of the calculus type:

• It is a combination of computer work and calculating on paper, which is time-consuming for the student.

Reasoning:

Questions with a choice of one or more answers can be selected for this type. Questions with the choice of more answers look more closely into the ability of students to use theory in solving tasks.

The following exercise is an example of choosing one answer.

Example D:

Find correct assertion about the following system of linear equations $x + 2y + 3z = 4$, $2x + y - z = 3$, $x + 2y + 3z = 7$.

Choose the answer: a) system has solution $x = \frac{2}{3}, y = \frac{5}{3}, y = 0$, b) system has no solution, c) no of the other answers is correct, d) system has infinite number of solutions.

It is obvious that the first and last equation cannot be both valid at the same time, the system therefore has no solution, answer b). The right answer can be found by simple reasoning or knowledge of the Frobenius theorem.

Other exercises have more answers. Students do not know in advance how many answers are correct.

Example E:

Sign all correct assertions about the system of linear equations $x_1 + 2x_2 + 3x_4 = 4,\ 2x_2 + x_3 - x_4 = 3$.

Choose the answers: a) system has only one solution, b) system has infinite number of solutions, c) no of the other answers is correct, d) system has no solution, e) system has solutions depending on two parameters.

It is clear that that the system has a solution (Frobenius theorem). There are two equations with four unknowns, ie. they depend on (4-2) parameters. Therefore, the only correct answers are b) and e).

Example F:

Calculate $\int \sin x \cos x \, dx$.

Choose the answers: a) $-\frac{(\cos x)^2}{2} + C, C \in R$, b) $-\frac{1}{4}\cos 2x + C, C \in R$, c) $\frac{(\sin x)^2}{2} + C, C \in R$, d) 5.

Knowledge of derivatives of composite functions and the properties of trigonometric functions show us that a), b) and c) are correct. This is only a different transliteration of the same function.

Benefits of reasoning:

• verifying the application of theoretical knowledge on a specific example,
• the use of this type is very suitable as a control of understanding theory
• suitable for continuous monitoring of students' knowledge,
• test runs quickly if the student understands theory very well,
• quick computer evaluation, saving time for the teacher.

Disadvantages of reasoning:

• in case of misunderstanding theory the test is very difficult and time consuming for the student.
• in case that the test with multiple choice was written, test evaluation by the teacher is more demanding compared to the test with one answer.

Note that testing using closed questions does not need to be necessarily electronic. Even paper tests lead to significant time savings (Chládek et al (2011)).

Conclusions

Our experience with electronic testing (approximately 600 persons) can be evaluated as positive. It is necessary to realize the purpose for which the students are tested. Therefore, it is necessary to choose the types of examples used for testing very carefully. We can easily test different skill than originally planned (comment to Example B).

The application of knowledge of specific calculations and application of theory in calculating the examples can be tested. Testing theoretical knowledge can also be carried out without problems, even though we have not discussed this possibility.

The benefits of testing dominate in testing partial knowledge, with large groups of people and during limited time. Computer-adaptive testing is very suitable for continuous testing of mathematical knowledge, see Chládek et al. (2011) Chládek & Smetanová(2014) Chládek & Smetanová (2015).

In terms of a thorough and detailed testing mathematical knowledge is the electronic testing disadvantageous. We are unable to determine students' way of thinking and the process of solving the exercises, if the students actually use the procedures anticipated, or something else, which we see as a great disadvantage. Therefore, we recommend for examination tests classical written form, which shows the calculation procedure and students' way of thinking.

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